Advances in Science and Technology, Vol. 11, No. 2, September / October 2017, pp 85 - 93

Numerical Modeling of Electronic Rectifier Using Differential Equations

Ikechukwu B. Ijeh* and Ezekiel C. Ede
Department of Physics, COLPAS, Michael Okpara University of Agriculture, Umudike,
PMB 7267, Umuahia, Abia State, Nigeria.

(Submitted: August 13, 2017; Accepted: October 17, 2017)

Abstract

Rectifier circuits are prevalent in modern electronic equipment. These circuits convert the alternating current into the direct current used by many devices. This work identifies and analyzes some of these circuits. For each circuit, a system of ordinary differential equations was developed using Kirchhoff laws, Ohm's law and diode law (Shockley equation). For each differential equation, a simple MATLAB Code was developed and numerical solution obtained using Runge kutta method (fourth order). The numerical solutions were obtained at various initial and boundary conditions with time ranging from 0 to 0.2sec at step height of 0.01. Based on this twenty one (21) iterations were performed according to twenty one (21) iterates. This numerical data is well supported with asymptotic solution obtained using averaging and the method of multiple scales. The graph of V(t) against Time for each circuit was obtained which resembles the saw tooth pictures associated with electronic rectifiers.

Keywords: Alternating current, differential equation, Kirchoff's laws, numerical, Rectifier

1.0 Introduction

Singh (2010) defined modeling as the study of processes and objects in another physical environment as models that duplicate the behaviour of the systems under observations. Modeling is an effective tool that can be easily applied to any field of study. Eykhoff (2002) defined a mathematical model as 'a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form'.

Bellomo and Preziosi (1995) defined mathematical model to be a set of equations which can be used to compute the time-space evolution of a physical system. Bender (2011) defined a mathematical model as "an abstract, simplified, mathematical construct related to a part of reality and created for a particular purpose."

There are four levels of modelling single phased diode rectifier. The first level of modeling is the ideal one. In this model, the diode is assumed to be ideal switch and the source resistance and inductance are neglected. The model has a very limited accuracy, but on the other hand nearly no parameters are needed. The second level is a table-based model of diode rectifier. The inaccuracy obtained using this simple approach may be justified by the fact that only a limited number of parameters need to be known. Recognizing that in a practical application all system parameters are difficult to obtain one can claim that this approach is well suited for calculation of the harmonic distortion in practical applications. The third level of the diode rectifier model is based on analytical model (Hussein *et al*., 2007).

The analytical model and especially in the discontinuous conduction mode is quite well described in the literature (Mansoor *et al*., 1995; Mansoor *et al*., 2003; Carpinelli *et al*., 2003; The fourth level of the diode rectifier model is the use of numerical based simulators that are non-linear components, such as diodes.

Mathematical modeling problems are often classified into black box or white box models, according to how much a priori information is available in the system. A black-box model is a system of which there is no priori information available. A white-box model (also called glass box or clear box) is a system where all necessary information is available.

***Corresponding Author's E-mail Address: bonijeh@gmail.com**

Practically all systems are somewhere between the black-box and white-box models, so this concept only works as an intuitive guide for approach. This work covers the development of mathematical model of electronic rectifier using differential equation, using a white box mathematical modeling approach.

Mathematically, the behavior of analog circuits can be described by continuous variables and a set of differential equations, whereas discrete variables and switching-modes are also used for modeling the mixed-signal circuits. Thereby, analog and mixed-signal circuits are hybrid system in nature.The analysis of continuous and mixed discrete-continuous systems is inherently difficult and many different abstractions in combination with dedicated verification techniques are currently being investigated by researchers. Several attempts have been made to apply the formal verification techniques of hybrid systems in the context of the formal verification of analog and mixed-signal circuits (Zaki *et al*., 2008; Scott *et al*., 2007; Gupta *et al*., 2004; Hartong *et al*., 2002).

2.0 Methods of Numerical Solutions of Differential Equations

The range of differential equations that can be solved by straightforward analytical methods is relatively restricted. Series solutions may not always be satisfactory, because of the involved manipulation in repeated stages of differentiation. In such cases, where differential equation and known boundary conditions are given, an approximate solution is often obtainable by the application of numerical methods, where a numerical solution is obtained at discrete values of the independent variable. In this work, three numerical methods of solving differential equations are considered.

2.1.1 Taylor's Expansion Series Method

Given an initial valued problem

$$\frac{dy}{dx} = \mathrm{f(x,y)}, \qquad \mathrm{y(x_o)} = \mathrm{y_o}$$

where *f* is a function of two variables and (x_o, y_o) is a single given point through which the solution curve passes, we assume that the various partial derivatives of the solution y exist. We then obtain the Taylor's series for *y* written as;

$$\mathrm{y(x+h)} = \mathrm{y(x)} + \mathrm{hy'(x)} + \frac{h^2}{2!}\mathrm{y''(x)} + \frac{h^3}{3!}\mathrm{y'''(x)} + \cdots + \frac{h^n}{n!}\mathrm{y^n(x)}$$

where *h* is the step size and $y' \equiv \frac{dy}{dx}$

2.1.2 Euler's Method

Given an initial valued problem

$$\mathrm{y^1} = \mathrm{f(x,y), y(x_o) = y_o}$$

then the Euler's method is given as**;**

$$y_{(n+1)} = y_n + hf(x_o, y_o), n = 0,1,2 \ldots$$

where *h* = step size and *n* is any integer number

2.1.3 Runge-Kutta Method

Runge-Kutta method was developed from the collective efforts of two mathematicians C. Runge and W. Kutta (see Butcher,1996). It is a single step that involves Taylor's series expansion but avoids the inconvenience of successive differentiation.

Given an initial valued problem

$$y^1 = f(x,y), y(x_0) = y_0,$$

The second order Runge-Kutta method is given by;

$$\boldsymbol{y_{n+1} = y(x_n) + ½(F_1 + F_2)};$$

where $\boldsymbol{F_1 = hf(x_n, y_n)}$;

$$\boldsymbol{F_2 = hf(x_n + h, y_n + F_1)}$$

The third order Runge-Kutta method is given by;

$$\boldsymbol{y_{n+1} = y(x_n) + \frac{1}{9}(2F_1 + 3F_2 + 4F_3)}$$

where $\boldsymbol{F_1 = hf(x_n + y_n)}$

$$\boldsymbol{F_2 = hf(x_n + ½h, y_n + ½F_1)}$$

$$\boldsymbol{F_3 = hf(x_n + \tfrac{3}{4}h, y_n + \tfrac{3}{4}F_2)}$$

The fourth order Runge- Kutta method is given by,

$$\boldsymbol{y_{n+1} = y(x_n) + \frac{1}{6}(F_1 + 2F_2 + 2F_3 + F_4)}$$

where

$$\boldsymbol{F_1 = hf(x_n, y_n)},$$

$$\boldsymbol{F_2 = hf(x_n + ½h, y_n + ½F_1)}$$

$$\boldsymbol{F_3 = hf(x_n + ½h, y_n + ½F_2)}$$

$$\boldsymbol{F_4 = hf(x + h, y + F_3)}$$

2.2 Basic Laws Governing the Flow of Current in a Circuit

The following laws govern the flow of current in a circuit:

2.2.1 Kirchhoff's Laws

The Kirchhoff's laws are divided into two namely; Kirchhoff's voltage law: This states that the algebraic sum of all the voltages around any closed loop in a circuit is zero.$\sum V = 0$, where V is the voltage. Kirchhoff's current law: The Kirchhoff current law states that the algebraic sum of the current in a given node is zero. Therefore$\sum I = 0$, where *I* = current.

2.2.2 Ohm's Law

Ohm's law states that the current that passes through a metallic conductor is directly proportional to the potential difference between its ends provided temperature and other physical factors remain constant. Therefore $V=IR$
where V is Voltage, I is Current and R is resistance.

2.2.3 Diode Law

The Shockley ideal diode equation or the diode law gives the I–V characteristics of an ideal diode in either forward or reverse bias (or no bias). The following equation is called the Shockley ideal diode equation where n the ideality factor is set equal to 1:

$$I = I_S\left(e^{V_D/(nV_T)} - 1\right)$$

where I is the diode current, I_S is the reverse bias saturation current (or scale current), V_D is the voltage across the diode, V_T is the thermal voltage, and n is the ideality factor also known as the quality factor or sometimes emission coefficient. The ideality factor n typically varies from 1 to 2 (though can in some cases be higher).

2.3 Rectifier Circuits

Rectifier circuits may be single-phase or multi-phase (three being the most common number of phases). Most low power rectifiers for domestic equipment are single-phase, but three-phase rectification is very important for industrial applications and for the transmission of energy as DC (HVDC).

2.3.1 Full-Wave Rectifier Circuit

In Full Wave Rectifier circuit two diodes are now used, one for each half of the cycle. A multiple winding transformer is used whose secondary winding is split equally into two halves with a common centre tapped connection, (C). This configuration results in each diode conducting in turn when its anode terminal is positive with respect to the transformer centre point C producing an output during both half-cycles twice that for the half wave rectifier.

2.3.2 Full Wave Rectifier Circuit

The full wave rectifier circuit consists of two power diodes connected to a single load resistance (R_L) with each diode supplying current to the load shown in Figure 1. When point A of the transformer is positive with respect to point C, diode D_1 conducts in the forward direction as indicated by the arrows. When point B is positive (in the negative half of the cycle) with respect to point C, diode D_2 conducts in the forward direction and the current flowing through resistor R is in the same direction for both half-cycles. As the output voltage across the resistor R is the phasor sum of the two waveforms combined, this type of full wave rectifier circuit is also known as a "bi-phase" circuit.

As the spaces between each half-wave developed by each diode is now being filled in by the other diode the average DC output voltage across the load resistor is now double that of the single half-wave rectifier circuit and is about $0.637V_{max}$ of the peak voltage, assuming no losses.

The peak voltage of the output waveform is the same as before for the half-wave rectifier provided each half of the transformer windings have the same rms voltage value. To obtain a different DC voltage output different transformer ratios can be used. The main disadvantage of this type of full wave rectifier circuit is that a larger transformer for a given power output is required with two separate but identical secondary windings making this type of full wave rectifying circuit costly compared to the "Full Wave Bridge Rectifier" circuit equivalent.

2.3.3 The Full Wave Bridge Rectifier

Another type of circuit that produces the same output waveform as the full wave rectifier circuit above is that of the Full Wave Bridge Rectifier. This type of single phase rectifier uses four individual rectifying diodes connected in a closed loop "bridge" configuration to produce the desired output. The main advantage of this bridge circuit is that it does not require a special centre tapped transformer, thereby reducing its size and cost. The single secondary winding is connected to one side of the diode bridge network and the load to the other side as shown in Figure 2.

2.3.4 The Diode Bridge Rectifier

The four diodes labelled D_1 to D_4 are arranged in "series pairs" with only two diodes conducting current during each half cycle. During the positive half cycle of the supply, diodes D1 and D2 conduct in series while diodes D3 and D4 are reverse biased and the current flows through the load as shown in

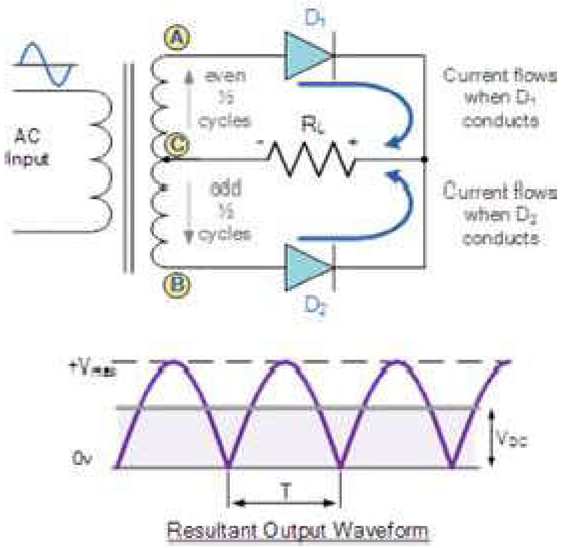

Figure 1: Full wave rectifier circuit.

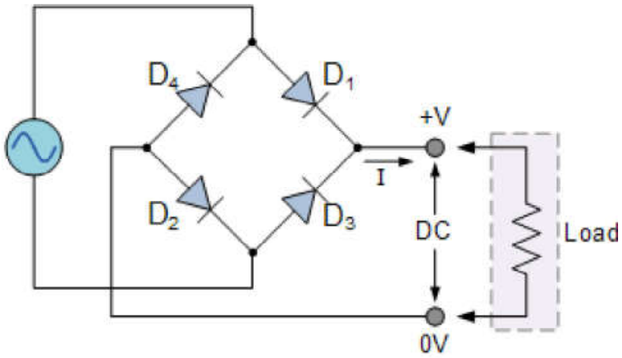

Figure 2: Full wave bridge rectifier circuit.

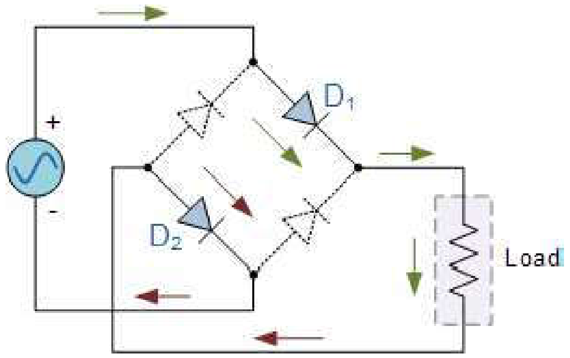

Figure 3: The Positive Half-cycle.

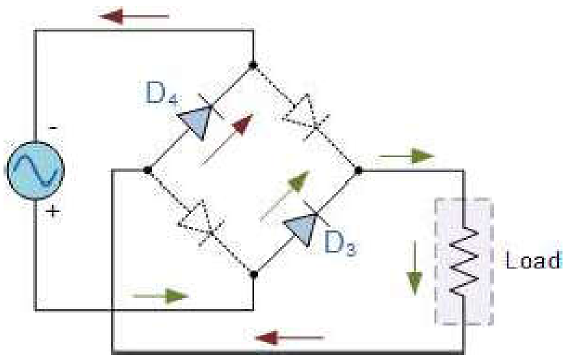

Figure 4: The Negative Half-cycle

Figure 3. During the negative half cycle of the supply, diodes D3 and D4 conduct in series, but diodes D1 and D2 switch "OFF" as they are now reversed biased, shown in Figure 4. The current flowing through the load is in the same direction as before.

As the current flowing through the load is unidirectional, so the voltage developed across the load is also unidirectional the same as for the previous two diode full-wave rectifier, therefore the average DC voltage across the load is $0.637V_{max}$.

2.3.5 Full-Wave Rectifier with Smoothing Capacitor

It is observed that the single phase half-wave rectifier produces an output wave every half cycle and that it was not practical to use this type of circuit to produce a steady DC supply. The full-wave bridge rectifier however, gives us a greater mean DC value (0.637V max) with less superimposed ripple while the output waveform is twice that of the frequency of the input supply frequency. We can therefore increase its average DC output level even higher by connecting a suitable smoothing capacitor across the output of the bridge circuit as shown in Figure 5.

2.3.6 Full-wave Rectifier with Smoothing Capacitor

The smoothing capacitor converts the full-wave rippled output of the rectifier into a smooth DC output voltage. Generally for DC power supply circuits the smoothing capacitor is an Aluminium Electrolytic type that has a capacitance value of 100μF or more with repeated DC voltage pulses from the rectifier charging up the capacitor to peak voltage. However, there are two important parameters to consider when choosing a suitable smoothing capacitor and these are its Working Voltage, which must be higher than the load output value of the rectifier and its Capacitance Value, which determines the amount of ripple that will appear superimposed on top of the DC voltage.

Too low a capacitance value and the capacitor has little effect on the output waveform. But if the smoothing capacitor is sufficiently large enough (parallel capacitors can be used) and the load current is not too large, the output voltage will be almost as smooth as pure DC. As a general rule of thumb, we are looking to have a ripple voltage of less than

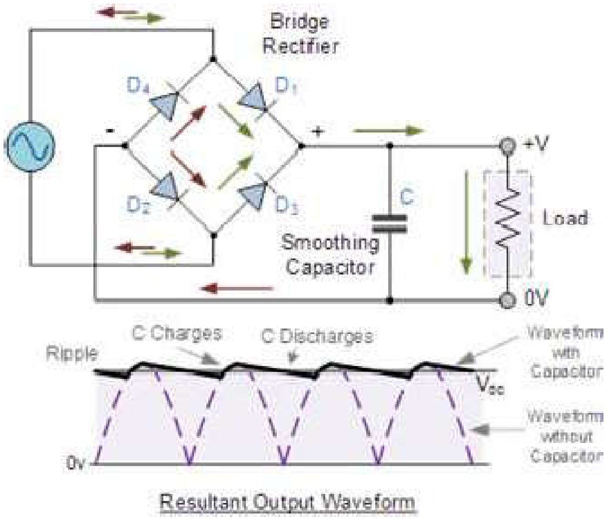

Figure 5: Full-wave Rectifier with Smoothing Capacitor.

100mV peak to peak.

The maximum ripple voltage present for a Full Wave Rectifier circuit is not only determined by the value of the smoothing capacitor but by the frequency and load current, and is calculated as:

$$V_{(ripple)} = \frac{I}{fC}$$

where *I* is the DC load current in amps, *f* is the frequency of the ripple or twice the input frequency in Hertz, and *C* is the capacitance in Farads. The main advantages of a full-wave bridge rectifier is that it has a smaller AC ripple value for a given load and a smaller reservoir or smoothing capacitor than an equivalent half-wave rectifier. Therefore, the fundamental frequency of the ripple voltage is twice that of the AC supply frequency (100Hz) where for the half-wave rectifier it is exactly equal to the supply frequency (50Hz).

The amount of ripple voltage that is superimposed on top of the DC supply voltage by the diodes can be virtually eliminated by adding a much improved ð-filter (pi-filter) to the output terminals of the bridge rectifier. This type of low-pass filter consists of two smoothing capacitors, usually of the same value and a choke or inductance across them to introduce a high impedance path to the alternating ripple component.

3.0 Data Analysis and Interpretation

A simple MATLAB code for each equation is first developed and the various iterates for the respective equations are obtained. A plot of voltage V(t) against the time is also obtained for each equation.

$$\frac{dV}{dt} = (\exp^{40\sin wt - V} - 1) - 10^{-4}V,$$

$$V(0) = 0,\ w(0) = 30,\ 0 \leq t \leq 20,\ h = 0.01$$

The iterates are stated in Table 1.

Table 1: Iterated voltages for Half Wave Rectifier Circuit.

n	Time (s)	F1	F2	F3	F4	V(v)
01	0.00	0.000000000	0.00110399	0.00086150	0.00191220	0.00097387
02	0.01	0.00185961	0.00268853	0.00247991	0.00324268	0.00354706
03	0.02	0.00319520	0.00374577	0.00359524	0.00409195	0.00720859
04	0.03	0.00405758	0.00439411	0.00429756	0.00459783	0.01154840
05	0.04	0.00457726	0.00477225	0.00471475	0.00488765	0.01628820
06	0.05	0.00487792	0.00498687	0.00495425	0.00505047	0.02125660
07	0.06	0.00504823	0.00510715	0.00508936	0.00514123	0.02635370
08	0.07	0.00514395	0.00517441	0.00516517	0.00519186	0.03152290
09	0.08	0.00519791	0.00521224	0.00520788	0.00522032	0.03673260
10	0.09	0.00522874	0.00523384	0.00523229	0.00523657	0.04196560
11	0.10	0.00524681	0.00524651	0.00524660	0.00524610	0.04721210
12	0.11	0.00525784	0.00525423	0.00525533	0.00525189	0.05246690
13	0.12	0.00526497	0.00525918	0.00526095	0.00525558	0.05772700
14	0.13	0.00526990	0.00526257	0.00526481	0.00525806	0.06299080
15	0.14	0.00527356	0.00526503	0.00526764	0.00525981	0.06825730
16	0.15	0.00527647	0.00526693	0.00526984	0.00526109	0.07352580
17	0.16	0.00527890	0.00526844	0.00527163	0.00526205	0.07879590
18	0.17	0.00528099	0.00526967	0.00527313	0.00526277	0.08406750
19	0.18	0.00528284	0.00527069	0.00527440	0.00526329	0.08934020
20	0.19	0.00528447	0.00527151	0.00527547	0.00526362	0.09461390

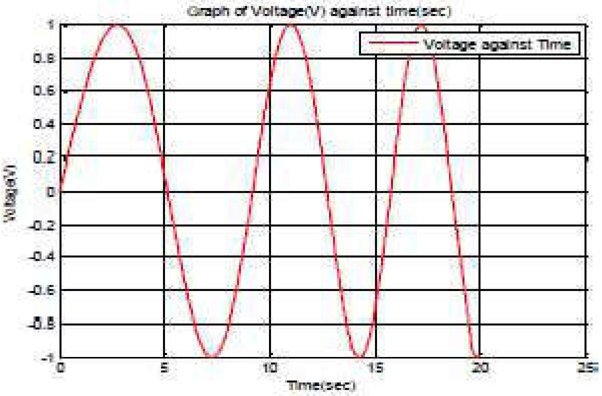

Figure 6: Plot of voltage against time for Half Wave Rectifier Circuit.

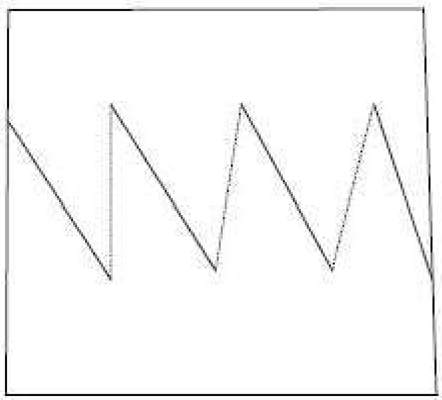

Figure 7: Oscillation in the tail of V(t)

Therefore, upon closer examination of Figure 6 as it approaches its asymptote it shows that the curve is not straight but oscillates as depicted in Figure 7. This resembles the saw tooth picture associated with electronic rectifiers.

$$\frac{dV}{dt} = 40\,(\sin(wt) - 1) - 10^{-4}\,v;\ (0) = 0;$$

$$w(0) = 30;\ 0 \le t \le 20,\ h=0.01$$

The iterates are indicated in Table 2.

Therefore, upon closer examination of Figure 8 the curve is not straight but oscillates as depicted in Figure 7. These resemble the saw tooth pictures associated with rectifiers. Also Figure 8 shows that the addition of another resistor before the diode is in order to model energy loss in the transformer. Thus it was clearly observed that energy is lost by the transformer in form of heat and eddy current.

Table 2: Iterated values of voltages for Half Wave Rectifier circuit with extra Resistor

n	Time (s)	F1	F2	F3	F4	V (v)
01	0.00	-0.336580	-0.336021	-0.337021	-0.335452	-0.336021
02	0.01	-0.335451	-0.334880	-0.335880	-0.334305	-0.670900
03	0.02	-0.334305	-0.337280	-0.338280	-0.333149	-1.004630
04	0.03	-0.331480	-0.332566	-0.333566	-0.331982	-1.337190
05	0.04	-0.331981	-0.331395	-0.332395	-0.330806	-1.668590
06	0.05	-0.330804	-0.330213	-0.331213	-0.329620	-1.998800
07	0.06	-0.329618	-0.329022	-0.329122	-0.328424	-2.327820
08	0.07	-0.328421	-0.327821	-0.328821	-0.327218	-2.655640
09	0.08	-0.327215	-0.326609	-0.327609	-0.326002	-2.982250
10	0.09	-0.325998	-0.325388	-0.326388	-0.324776	-3.307640
11	0.10	-0.324772	-0.324158	-0.325158	-0.323541	-3.631800
12	0.11	-0.323536	-0.322917	-0.323917	-0.322295	-3.954710
13	0.12	-0.322290	-0.321666	-0.322666	-0.321040	-4.276380
14	0.13	-0.321035	-0.320406	-0.321406	-0.319775	-4.596780
15	0.14	-0.319769	-0.319136	-0.319236	-0.318501	-4.915920
16	0.15	-0.318494	-0.317856	-0.318856	-0.317216	-5.233780
17	0.16	-0.317209	-0.316567	-0.317567	-0.315922	-5.550340
18	0.17	-0.315915	-0.315267	-0.316267	-0.314618	-5.865610
19	0.18	-0.314610	-0.313959	-0.314959	-0.313305	-6.179570
20	0.19	-0.313296	-0.31264	-0.31364	-0.311981	-6.49221

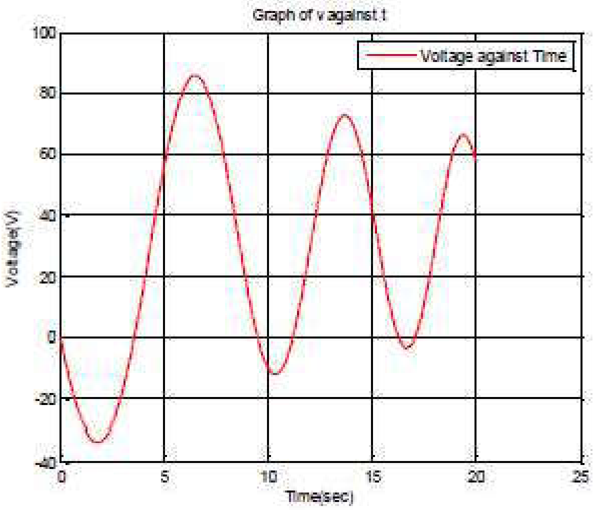

Figure 8: Plot of voltage verses time for Half Wave Rectifier with extra Resistor.

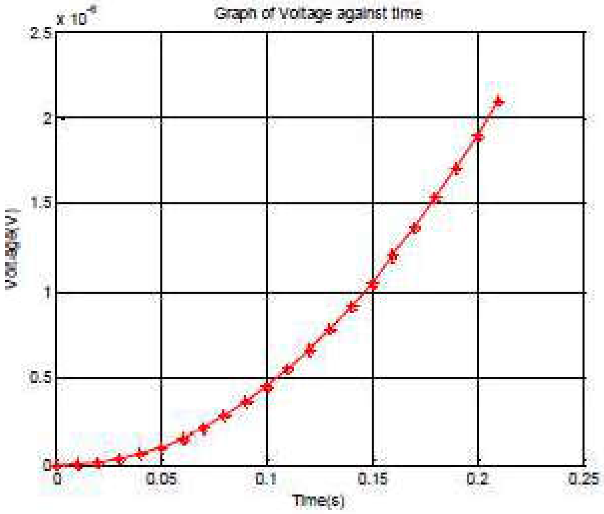

Figure 9: Graph of Voltage against Time for Full Wave Rectifier.

$$\frac{dV}{dt} = 10^{-4} \{i + 2e^{-40V}\cosh(40\sin t) - 2\},$$
$$i(0) = 0,\ V(0) = 0,\ 0 \le t \le 20,\ h = 0.01$$

The iterates shown in Table 3 were obtained.

Figure 9 shows that in the full wave rectifier circuit, the ripple is significantly lower compared to the half wave rectifier. This is as a result of the additional diode and filtering components. Also Figure 9 shows that by adding the extra capacitor and inductor, we have eliminated the ripple in the output voltage and have reduced the amplitude of the ripple of the voltage across the diode.

Figure 10 is a plot of Voltage against the Current. Upon closer examination of this figure, the plot is not straight but scattered. This implies that diodes are non-Ohmic devices.

In Figure 11, we notice that while the steady state

Table 3: Iterated voltages for Full Wave rectifier

n	T (s)	F1	F2	F3	F4	V (V)	I (A)
1	0.00	0.00000	3.71167e-15	3.71152e-15	1.48464e-14	4.9488e-15	0.000
2	0.01	1e-08	9.99963e-09	9.99963e-09	9.99926e-09	9.99964e-09	0.013
3	0.02	1.99993e-08	1.99985e-08	1.99985e-08	1.99977e-08	2.99981e-08	0.026
4	0.03	2.99977e-08	2.99966e-08	2.99966e-08	2.99954e-08	5.99947e-08	0.031
5	0.04	3.99954e-08	3.99939e-08	3.99939e-08	3.99924e-08	9.99886e-08	0.040
6	0.05	4.99924e-08	4.99905e-08	4.99905e-08	4.99885e-08	1.49979e-07	0.051
7	0.06	5.99885e-08	5.99862e-08	5.99862e-08	5.99839e-08	2.09965e-07	0.068
8	0.07	6.99839e-08	6.99812e-08	6.99812e-08	6.99786e-08	2.79947e-07	0.710
9	0.08	7.99786e-08	7.99755e-08	7.99755e-08	7.99724e-08	3.59922e-07	0.800
10	0.09	8.99724e-08	8.99689e-08	8.99689e-08	8.99655e-08	4.49891e-07	0.920
11	0.10	9.99655e-08	9.99616e-08	9.99616e-08	9.99578e-08	5.49853e-07	1.000
12	0.11	1.09958e-07	1.09954e-07	1.09954e-07	1.09949e-07	6.59806e-07	1.020
13	0.12	1.19949e-07	1.19945e-07	1.19945e-07	1.1994e-07	7.79751e-07	1.970
14	0.13	1.2994e-07	1.29935e-07	1.29935e-07	1.2993e-07	9.09686e-07	2.450
15	0.14	1.3993e-07	1.39925e-07	1.39925e-07	1.39919e-07	1.04961e-06	2.690
16	0.15	1.49919e-07	1.49914e-07	1.49914e-07	1.49908e-07	1.19952e-06	3.010
17	0.16	1.59908e-07	1.59902e-07	1.59902e-07	1.59896e-07	1.35943e-06	4.670
18	0.17	1.69896e-07	1.69889e-07	1.69889e-07	1.69882e-07	1.52932e-06	4.720
19	0.18	1.79882e-07	1.79876e-07	1.79876e-07	1.79869e-07	1.70919e-06	4.890
20	0.19	1.89869e-07	1.89861e-07	1.89861e-07	1.89854e-07	1.89905e-06	5.070

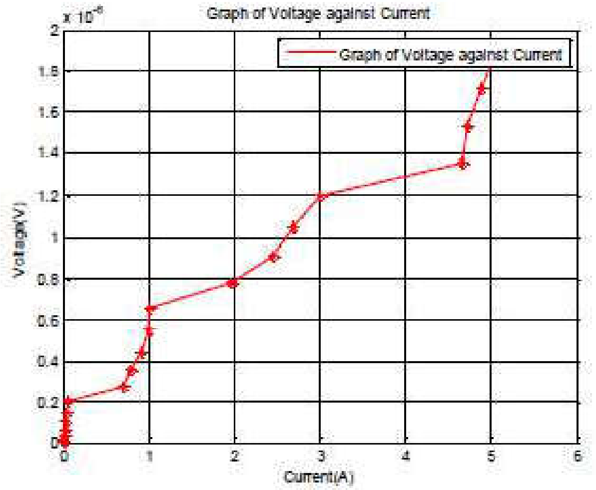

Figure 10: Plot of Voltage against Current.

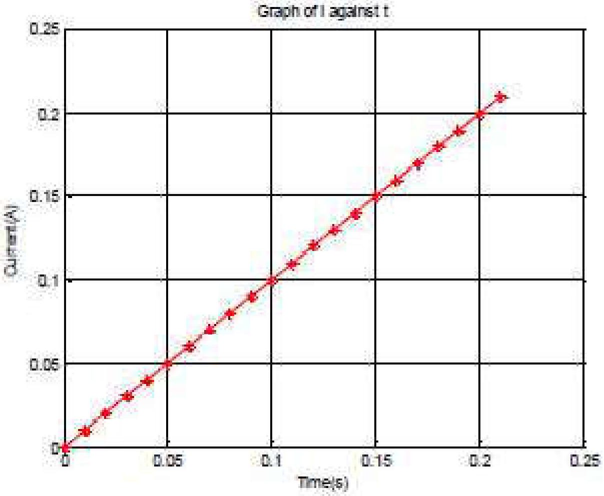

Figure 11: Graph of current against time

is reached very slowly, there are no visible oscillations. Meanwhile, current (i) achieved its steady state without many visible oscillation.

4.0 Conclusion and Recommendation

The necessity of modeling lies in the nature of technology and its advancement. The modeling minimizes time and cost of the process involved. The mathematical model provides an insight into the behavior of the physical system that reduces the problem to its essential characteristics. The glass box approach is an elegant method of mathematical modeling of electronic devices and circuits. A rectifier is an electrical device that converts alternating current (AC), which periodically reverses direction to direct current (DC), which flows in only one direction. The process is known as rectification.

This research takes a look into mathematical modeling of electronic rectifier using differential equation. The glass box modelling approach has been used for modelling electronic devices. Some of our findings include:

i. diodes are passive electronic devices and are temperature dependent in accordance with diode law (Shockley equation).

ii. rectifiers have saw tooth picture and have periodic oscillation.

iv. rectifiers do not obey Ohm's laws and thus are said to be Non-Ohmic devices.

We recommend that mathematical modeling of electronic rectifier should be revisited using method of averaging and multiple scales to obtain the asymptotic solution. Also mathematical modeling should be used to probe various systems where direct experiments cannot be carried out.

References

Bender, E.A. 2011, An Introduction to Mathematical Modeling (NewYork: Dover Publications).

Butcher J. C. 1996, "A History of Runge-Kutta Methods", AppliedNumerical Methods, Elsevier **20**, 247-260.

Carpinelli, G., Iacovone, F., Varilone P. and Verde, P. 2003, "Single-Phase Voltage Source Converters: Analytical Modeling for Harmonic Analysis in Continuous and Discontinuous Current Conditions", International Journal of Power and Energy Systems, **23**, No. 1, 37-48.

Eykhoff, W. 2002, Rectifying Circuits. Power electronics (3rd ed. London: McGraw-Hill).

Gupta, S., Krogh, B.H. and Rutenbar, R.A. 2004, "Towards formal verification of analog designs", Proc. IEEE/ACM International Conference on Computer Aided Design (ICCAD-2004), Nov. 7-11, 2004, San Jose, CA (USA), 210 – 217.

Hartong, W., Hedrich, L. and Barke, E. 2002, "Model Checking Algorithms for Analog Verification", Proceedings of the ACM/IEEE DesignAutomation Conference, 2002, June 10-14, New Orleans, Louisiana. USA, 542-547.

Hussein A. K., Abdulhakeem A. A., Ali S. A., and Khamis H. A. 2007. Simple and Advanced Models for Calculating Single-Phase Diode Rectifier Line-Side Harmonics World Academy of Science, Engineering and Technology Conference, September 2007.

Mansoor, A., Grady, W. M., Chowdhury A. H., and Samotyj, M. J. 1995, "An Investigation of Harmonics Attenuation and Diversity Among Distributed Single-Phase Power Electronic Loads", IEEE Transactions on Power Delivery, **10**, No.1, 467-473.

Mansoor, A., Grady, W. M.,Thallam,R. S., Doyle,M. T., Krein S. D. and Samotyj, M. J. 2003, "Effect of Supply Voltage Harmonics on the Input Current of Single-Phase Diode bridge Rectifier Loads", IEEE, Transactions on Power Delivery, **10**, No. 3, 1416-1422.

Preziosi, L. and Bellomo, L.N. (1995) Modelling, Mathematical Methods and Scientific Computation, CRC Press.

Scott L., Walter, D., Jones K. and Cheris M, 2007, "Analog/Mixed-Signal Circuit Verification Using Models Generated from Simulation Traces", ATVA 2007, LNCS 4762, 114-128.

Singh, B.P. 2010, "Active Bridge for Measurement of the Admittance Parameters of the Transistor". Indian Journal of Pure & Applied Physics, **15**, 783-786.

Zaki, M. H., Tahar, S., and Bois, G. 2008, "Formal Verification of Analog and Mixed Signal Designs: A Survey", Microelectronics Journal, **39**, 1395 – 1404.